Orchestrating on-board sensors for global hybrid robust stabilization of unicycles

Philipp Braun

School of Engineering,
Australian National University, Canberra, Australia

In Collaboration with:

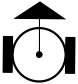
R. Ballaben: Faculty of Engineering, University of Porto (Portugal)

A. Astolfi: Applied Mathematics and Computational Science, King Abdullah University of Science and Technology (Saudi Arabia)

L. Zaccarian: Dipartimento di Ingegneria Industriale, University of Trento (Italy); and Laboratoire d'Analyse et d'Architecture des Systèmes-Centre Nationnal de la Recherche Scientifique, Université de Toulouse, CNRS (France)

Australian National University

- Introduction and background
  - Setting & Hybrid Systems & Lyapunov Stability
- Target stabilization
  - Unmanned Ground Vehicles (UGV)
- Recent Extensions
  - Unmanned Surface Vehicles (USV)



Ballaben, R., Astolfi, A., Braun, P., & Zaccarian, L. (2026b). *Orchestrating on-board sensors for global hybrid robust stabilization of unicycles* [Automatica]. https://hal.science/hal-05004452

What measurements can we use to close the loop?

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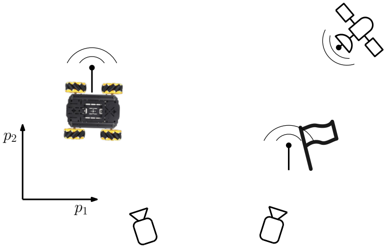

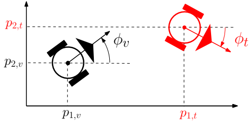

Cartesian coordinates

$$\begin{bmatrix} p_1 \\ p_2 \\ \phi \end{bmatrix} = \begin{bmatrix} p_{1,v} - p_{1,t} \\ p_{2,v} - p_{2,t} \\ \phi_v - \phi_t \end{bmatrix}$$

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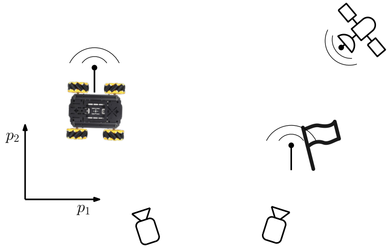

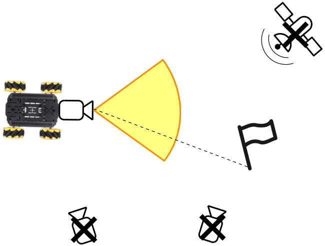

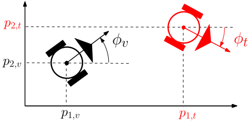

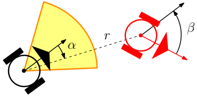

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Camera-based coordinates

$$z = \begin{bmatrix} r \\ \beta \\ \alpha \end{bmatrix}, \quad \alpha \in [-\overline{\alpha}, \overline{\alpha}]$$

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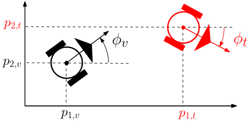

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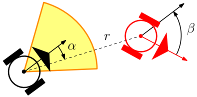
Coordinate transformation

$$\begin{bmatrix} p_1 \\ p_2 \\ \phi \end{bmatrix} = \gamma(z) := \begin{bmatrix} r\cos(\alpha - \beta - \pi) \\ r\sin(\alpha - \beta - \pi) \\ -\beta \end{bmatrix}$$

Stabilization using camera measurements

- If $(r, \alpha) \to 0$, then $(p_1, p_2) \to 0$
- Additionally, if $\beta \to 0$, then $\phi \to 0$



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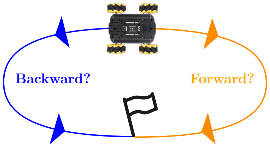
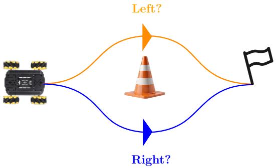
Decisions naturally arise when controlling a vehicle





Decisions are discontinuous control actions...

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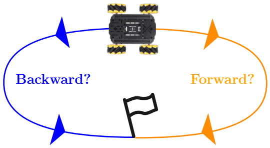

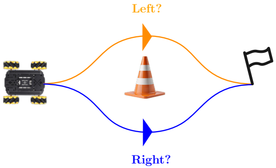

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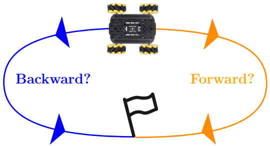

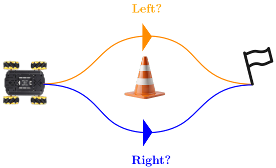

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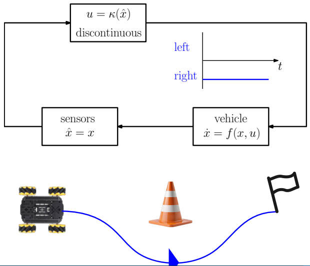

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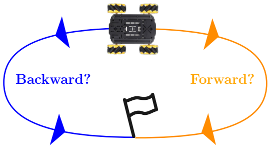

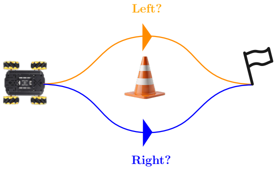

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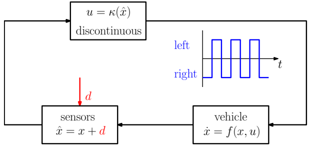

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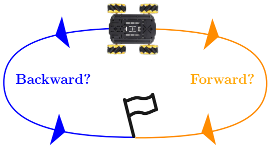

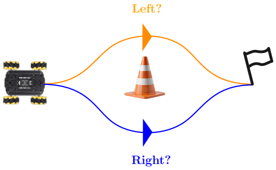

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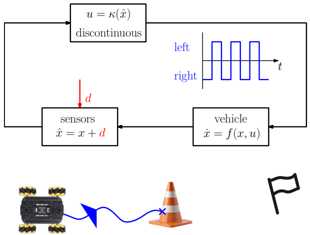
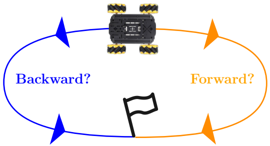
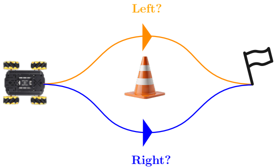


Discontinuous actions should be implemented in a **robust** way

Decisions naturally arise when controlling a vehicle





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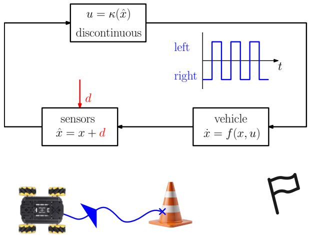


- Intrinsic limitations (Brockett, 1982)
- Using path-planning with tracking (Pathak & Agrawal, 2005)
- Discontinuous feedback (De Luca et al., 2000; Oelen et al., 1994) (σ-process coordinate change (Arnold, 1987))
- Time-invariant sample-and-hold (Astolfi, 1996, 1998, 1999),
- Discontinuous time-varying sample-and-hold (Braun et al., 2017; Pomet et al., 1992).
- Uniting local+global (Prieur, 2001; Prieur & Astolfi, 2003)
- Onboard sensors (Murrieri et al., 2004; Restrepo et al., 2021)

  Issues with angular winding

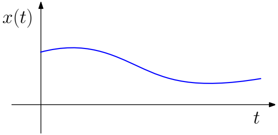
Discontinuous actions should be implemented in a **robust** way

Hybrid dynamical systems [Teel, Goebel, Sanfelice, 2012]: a tool for modeling discont. actions

Continuous-time systems

$$\dot{x} = f(x), \; x \in \mathbb{R}^n$$

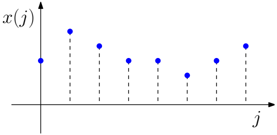
Model the dynamics of physical systems



Discrete-time systems

$$x^+ = g(x), \; x \in \mathbb{R}^n$$

Model instantaneous events



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Hybrid dynamical systems

$$\begin{cases} \dot{x} &= f(x), \quad x \in \mathcal{C}, \\ x^+ &= g(x), \quad x \in \mathcal{D}. \end{cases}$$

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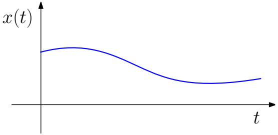
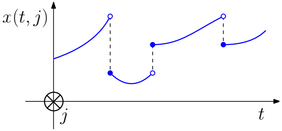


Combine both worlds



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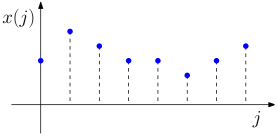

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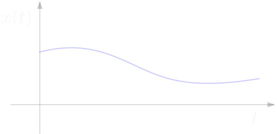

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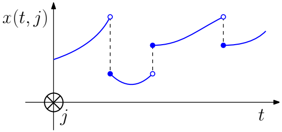

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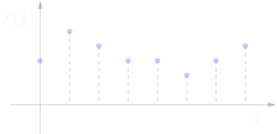

Terminology:

- f *flow map*
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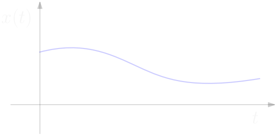

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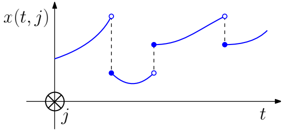

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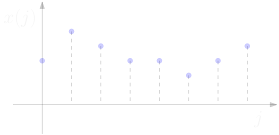

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Can model:

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$$x^+ = g(x), \; x \in \mathcal{D}$$

- Discontinuous actions

$q \in \{-1, 1\}$, q is a "logic" state variable

$$\dot{x} = f(x, q) = \begin{cases} f_+(x) & \text{if } q = 1, \\ f_-(x) & \text{if } q = -1. \end{cases}$$

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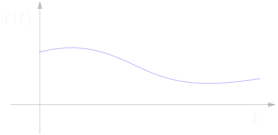

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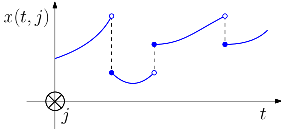

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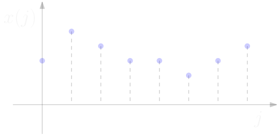

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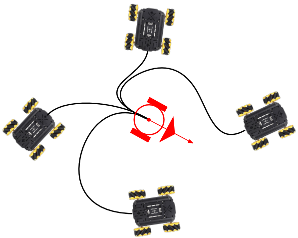
Design guidelines (Hybrid Basic Conditions)

- $\mathcal{C}$, $\mathcal{D}$ closed
- f, g continuous

The HBC guarantees **robustness** of asymptotic stability to perturbations and uncertainties



- Goal: stabilize fw invariant compact set $\mathcal{A}$
- Given the equations of dynamics of vehicle

$$\dot{x} = f(x, u)$$

- Design hybrid control architecture s.t.

$$(\mathcal{H}) \begin{cases} \dot{x} = f(x,u), & x \in \mathcal{C}, \\ u = \kappa(x), & \\ x^+ = g(x), & x \in \mathcal{D}, \end{cases}$$

$\mathcal{A}$ is GAS for $(\mathcal{H})$

Lyapunov-based control design methodology

Choose

- $u = \kappa(x)$, $\mathcal{C}$ — The control law driving the vehicle
- $g(x), \mathcal{D}$ — The discontinuous action, when it happens
- $V(x)$ positive definite, radially unbounded — The Lyapunov function candidate

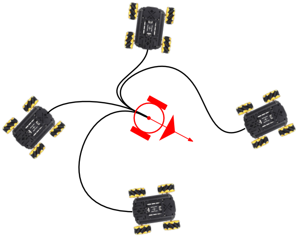
such that

$$\dot{V}(x) = \nabla V(x)^\top f(x, \kappa(x)) < 0, \quad \forall x \in \mathcal{C} \setminus \mathcal{A}$$
$$\Delta V(x) := V(g(x)) - V(x) < 0, \quad \forall x \in \mathcal{D} \setminus \mathcal{A}$$

V is decreasing along solutions to $(\mathcal{H})$

then, $\mathcal{A}$ is Globally Asymptotically Stable (GAS)

Lyapunov functions: a tool for control design



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V is **non-increasing (?)** along solutions to $(\mathcal{H})$

Behaviour of solutions can be exploited

then, $\mathcal{A}$ is Globally Asymptotically Stable (GAS)

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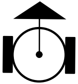
- Conclusions



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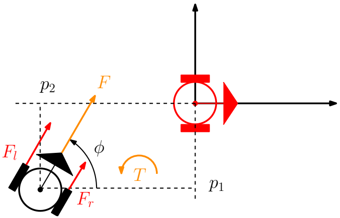
A kinematic model for UGV in the camera-based coordinates



Modeling assumptions

- F_r, F_l are independent
- F, T can be used to assign v, ω

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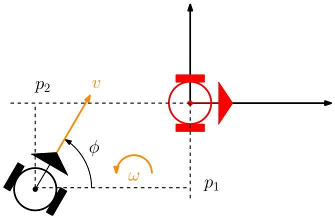

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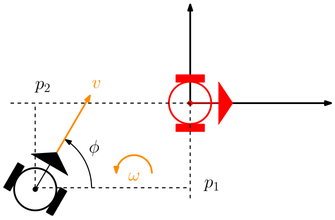
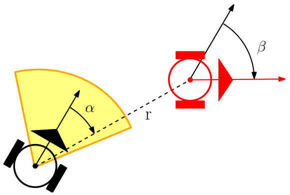
- Under the modeling assumptions, the motion of a UGV is described by the kinematic equations

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} v\cos(\phi) \\ v\sin(\phi) \\ \omega \end{bmatrix},$$

$$u = [v \ \omega]^\top$$





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$$u = [v \ \omega]^\top$$

- Exploiting the coordinate transformation $x = \gamma(z)$, the kinematic equations of a UGV in the camera-based coordinates are obtained

$$\begin{bmatrix} \dot{r} \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -\nu r\cos(\alpha) \\ -\omega \\ \nu\sin(\alpha) - \omega \end{bmatrix},$$

$$\alpha \in [-\overline{\alpha}, \overline{\alpha}], \quad u = [\nu \ \omega] = [\tfrac{v}{r} \ \omega]^\top$$

The measurements coming from one camera can be used to drive the vehicle to the target...

★ $\dot{z} = \begin{bmatrix} \dot{r} \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -\nu r \cos(\alpha) \\ -\omega \\ \nu \sin(\alpha) - \omega \end{bmatrix}, \qquad \alpha \in [-\overline{\alpha}, \overline{\alpha}], \ u = [\nu \ \omega] = [\frac{v}{r} \ \omega]$

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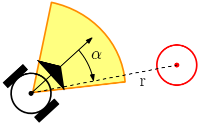
Camera measures (r, α)



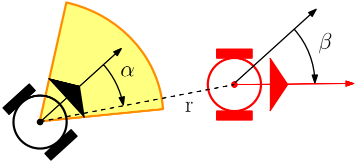
$$\kappa_0(r,\alpha) := \begin{bmatrix} k_r \cos(\alpha) \\ k_r \cos(\alpha)\sin(\alpha) + k_\alpha \alpha \end{bmatrix}$$

Theorem: Stabilization of target position

Let $k_r, k_\alpha \in \mathbb{R}_{>0}$ be arbitrary and $u = \kappa_0(r,\alpha)$. The set $\mathcal{A}_\phi = \{r = \alpha = 0\}$ is pre-asymptotically stable (pAS) for $\bigstar$.

$$V(z) = \frac{1}{2}(r^2 + \alpha^2)$$

Camera measures $z = (r, \beta, \alpha)$



$$\kappa(z) = \begin{bmatrix} k_r \cos(\alpha) \\ k_r \cos(\alpha)\sin(\alpha) + k_\alpha \alpha + k_\beta (\alpha - \beta) \frac{\cos(\alpha)\sin(\alpha)}{\alpha} \end{bmatrix}$$

Theorem: Stabilization of the target pose

Let $k_r, k_\beta, k_\alpha \in \mathbb{R}_{>0}$ be arbitrary and $u = \kappa(z)$. The origin $\mathcal{A} = (0,0,0)$ is pAS for $\bigstar$.

$$V(z) = \frac{1}{2}\left(r^2 + \frac{k_\beta}{k_r}(\alpha - \beta)^2 + \alpha^2\right)$$

The measurements coming from one camera can be used to drive the vehicle to the target...

$$\star \quad \dot{z} = \begin{bmatrix} \dot{r} \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -\nu r \cos(\alpha) \\ -\omega \\ \nu \sin(\alpha) - \omega \end{bmatrix}, \qquad \alpha \in [-\overline{\alpha}, \overline{\alpha}], \;\; u = [\nu \;\; \omega] = [\tfrac{v}{r} \;\; \omega]$$

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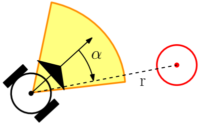

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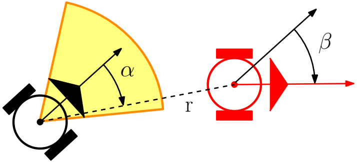

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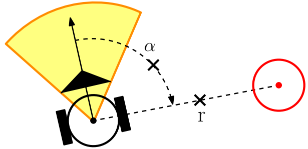
$$V(z) = \frac{1}{2}\left(r^2 + \frac{k_\beta}{k_r}(\alpha - \beta)^2 + \alpha^2\right)$$

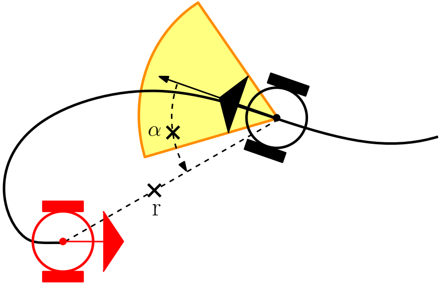
Solutions converge to the target if the target remains inside the field of view ($\alpha(t) \in [-\overline{\alpha}, \overline{\alpha}]$ for all t)

...but multiple cameras are required to achieve global stabilization



- The camera does not see the target for all possible configurations of the vehicle

$$\gamma(\mathcal{C} \cup \mathcal{D}) \neq \mathcal{X}$$



- The camera may lose vision of the target during the vehicle motion
- Not every solution to

$$\bigstar \; \dot{z} = f(z,u) := \begin{bmatrix} \dot{r} \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} -\nu r \cos(\alpha) \\ -\omega \\ \nu \sin(\alpha) - \omega \end{bmatrix},$$

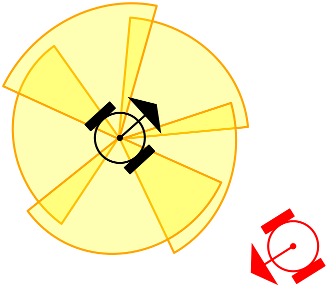
$$u = \kappa(z),$$

$$\alpha \in [-\overline{\alpha}, \overline{\alpha}]$$

is complete ($t \mapsto z(t)$ defined $\forall t \geq 0$)

Measurements from multiple cameras must be combined to globally stabilize the target

Multiple cameras can be combined into virtual front and rear camera, leading to forward and reverse motion



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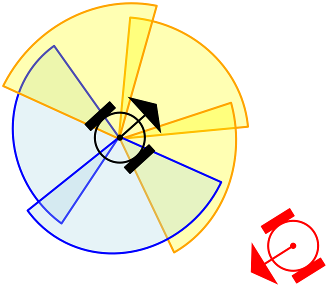

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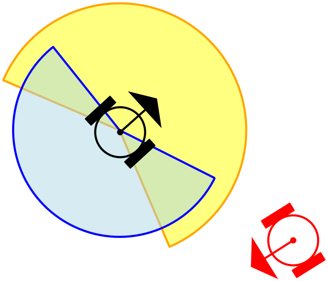
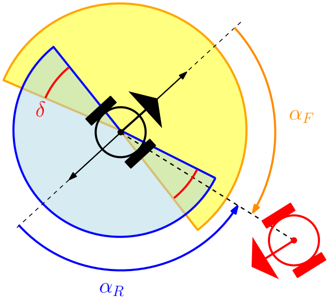


Multiple cameras can be combined into virtual front and rear camera, leading to forward and reverse motion

Two-cameras setup

Group cameras into a virtual front and rear camera

- Front camera measures (r, β, α_F) if $\alpha_F \in [-\frac{\pi}{2} - \delta, \frac{\pi}{2} + \delta]$
- Rear camera measures (r, β, α_R) if $\alpha_R \in [-\frac{\pi}{2} - \delta, \frac{\pi}{2} + \delta]$
- $\delta \in (0, \frac{\pi}{2})$ defines overlapping field of view of cameras



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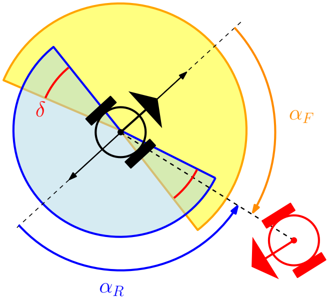
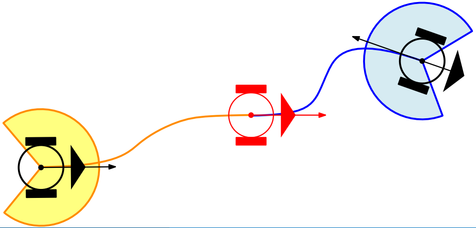
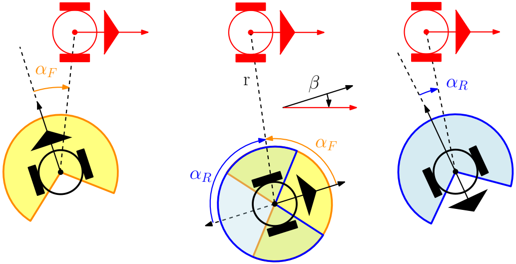


Discontinuous control action

- Front camera sees the target $\Rightarrow \alpha_F \to 0 \Rightarrow$ forward motion $(v_1 > 0)$
- Rear camera sees the target $\Rightarrow \alpha_R \to 0 \Rightarrow$ reverse motion $(v_1 < 0)$



The target can be stabilized using front and rear camera measurements



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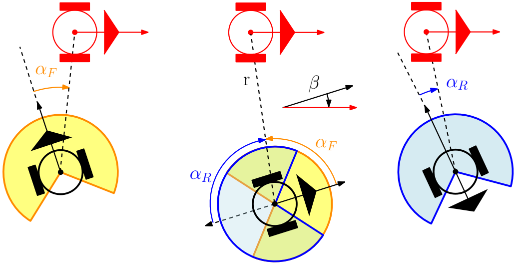

- Two sets of coordinates $z_F = (r, \beta, \alpha_F)$ and $z_R = (r, \beta, \alpha_R)$

$$\alpha_i \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta], \; i \in \{F, R\}, \quad \delta \in (0, \tfrac{\pi}{2})$$

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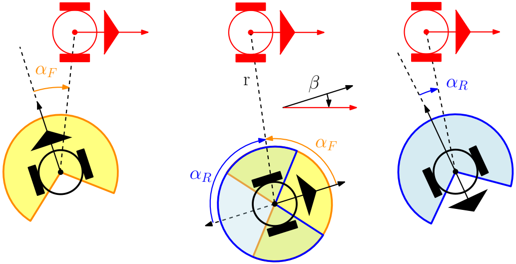

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$$\star \; \dot{z}_F = \begin{bmatrix} \dot{r} \\ \dot{\beta}_F \\ \dot{\alpha}_F \end{bmatrix} = \begin{bmatrix} -\nu r \cos(\alpha_F) \\ -\omega \\ +\nu \sin(\alpha_F) - \omega \end{bmatrix}$$

$$\alpha_F \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta]$$

$$\star \; \dot{z}_R = \begin{bmatrix} \dot{r} \\ \dot{\beta}_R \\ \dot{\alpha}_R \end{bmatrix} = \begin{bmatrix} +\nu r \cos(\alpha_R) \\ -\omega \\ -\nu \sin(\alpha_R) - \omega \end{bmatrix}$$

$$\alpha_R \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta]$$

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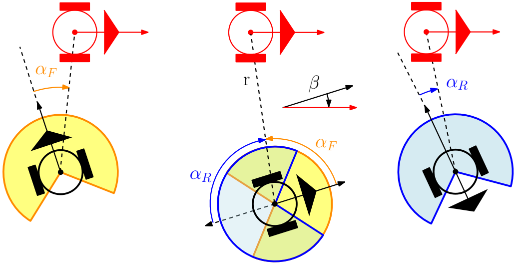

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$$\alpha_F \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta] \qquad \alpha_R \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta]$$

- Two stabilizers $i \in \{F, R\}$

Target position stabilizer

$$\kappa_0(z_F / z_R) = \begin{bmatrix} +/- k_r \cos(\alpha_i) \\ k_r \cos(\alpha_i)\sin(\alpha_i) + k_\alpha \alpha_i \end{bmatrix}$$

Target pose stabilizer

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A hybrid control architecture can combine front and rear camera measurements

- A logic variable $q \in \{-1, 1\}$ can be used to describe the camera used
- Hybrid state combines camera measurements with logic variable

$$\xi = (r, \beta, \alpha, q)$$

$$q = 1 \Rightarrow (r, \beta, \alpha) = z_{\mathrm{F}} \qquad q = -1 \Rightarrow (r, \beta, \alpha) = z_{\mathrm{R}}$$

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- Logic variable q unifies equations of motion and feedback selections into **flow dynamics**

$$\dot{\xi} = \begin{bmatrix} \dot{r} \\ \dot{\beta} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = f(\xi, u) := \begin{bmatrix} -q\nu r \cos(\alpha) \\ -\omega \\ q\nu \sin(\alpha) - \omega \\ 0 \end{bmatrix}, \xi \in \mathcal{C}, \quad u = \kappa(\xi) = \begin{bmatrix} qk_r \cos(\alpha) \\ k_r \cos(\alpha)\sin(\alpha) + k_\alpha \alpha + k_\beta (\alpha - \beta) \frac{\cos(\alpha)\sin(\alpha)}{\alpha} \end{bmatrix},$$

$$\mathcal{C} = \{\xi : |\alpha| \leq \tfrac{\pi}{2} + \delta\}$$

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- **Jump dynamics** defines the switch between cameras (the discontinuous action)

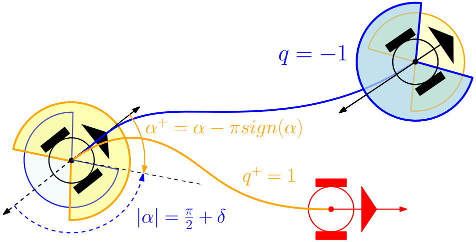
When cameras are switched

$$\xi \in \mathcal{D} := \{\xi : |\alpha| \geq \tfrac{\pi}{2} + \delta\}$$

How measurements change

$$\xi^+ = g(\xi) := \begin{bmatrix} r \\ \beta \\ \alpha - \pi \operatorname{sign}(\alpha) \\ -q \end{bmatrix}$$

$\delta \in (0, \frac{\pi}{2})$ ensures **hysteresis** in the camera switch



Reset of vehicle orientation

- Hybrid state combines camera measurements with logic variable

$$\xi = (r, \beta, \alpha, q), \quad \alpha \in [-\tfrac{\pi}{2} - \delta, \tfrac{\pi}{2} + \delta], \quad \beta \in [-\tfrac{3}{2}\pi - \delta, \tfrac{3}{2}\pi + \delta]$$

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$$\mathcal{C} = \{\xi : |\alpha| \leq \tfrac{\pi}{2} + \delta, \beta \in [-\tfrac{3}{2}\pi - \delta, \tfrac{3}{2}\pi + \delta]\}$$

- Jump dynamics

Camera switch

$$\mathcal{D}_\alpha := \{\xi \in \Xi : |\alpha| \geq \tfrac{\pi}{2} + \delta\}$$

$$g_\alpha(\xi) := \begin{bmatrix} r \\ \beta \\ \alpha - \pi \operatorname{sign}(\alpha) \\ -q \end{bmatrix}$$

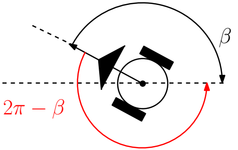
Orientation reset

$$\mathcal{D}_\beta := \{\xi \in \Xi : |\beta| \geq \tfrac{3}{2}\pi + \delta\}$$

$$g_\beta(\xi) := \begin{bmatrix} r \\ \beta - 2\pi \operatorname{sign}(\beta) \\ \alpha \\ q \end{bmatrix}$$

$$\xi^+ \in G(\xi) = \begin{cases} \{g_\beta(\xi)\}, & \text{if } \xi \in \mathcal{D}_\beta \backslash \mathcal{D}_\alpha, \\ \{g_\alpha(\xi)\}, & \text{if } \xi \in \mathcal{D}_\alpha \backslash \mathcal{D}_\beta, \\ \{g_\alpha(\xi)\} \cup \{g_\beta(\xi)\}, & \text{if } \xi \in \mathcal{D}_\alpha \cap \mathcal{D}_\beta, \end{cases}$$

$$\mathcal{D} := \mathcal{D}_\alpha \cup \mathcal{D}_\beta$$



Main result: global stabilization of target using camera measurements

- Closed-loop hybrid system

$$(\mathcal{H}) \left\{ \begin{array}{ll} \dot{\xi} = f(\xi, \kappa(\xi)), & \xi \in \mathcal{C}, \\ \xi^{+} \in g(\xi), & \xi \in \mathcal{D}. \end{array} \right.$$

- The Lyapunov function candidate

$$V(\xi) = \frac{1}{2}\left(r^2 + \frac{k_\beta}{k_r}(\alpha - \beta)^2 + \alpha^2\right)$$

is non-increasing when solutions flow ($\dot{V} \leq 0$), but **may increase when solutions jump** from $\mathcal{D}_\alpha$!

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Lemma: camera invariance

All solutions to $(\mathcal{H})$ jump from $\mathcal{D}_\alpha$ at most once.

Theorem: Target stabilization via two-cameras architecture

For any selection of control gains $k_r, k_\alpha \in \mathbb{R}_{>0}, k_\beta \in \mathbb{R}_{\geq 0}$ and $\delta \in (0, \frac{\pi}{2})$

- $\mathcal{A}_\phi = \{r = \alpha = 0\}$ is GAS for $(\mathcal{H})$
- If $k_\beta > 0$, $\mathcal{A} = \{r = \alpha = \beta = 0\}$ is GAS for $(\mathcal{H})$
- All solutions are complete

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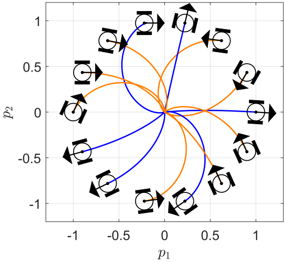

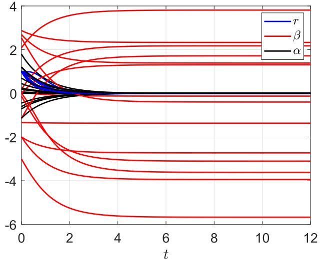

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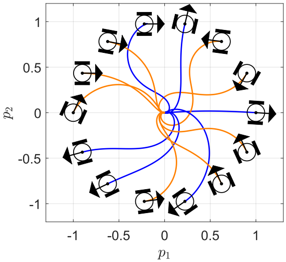

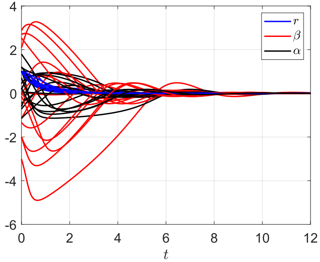

Robustness of GAS is guaranteed by hybrid well-posedness

$$(\mathcal{H}) \begin{cases} \dot{\xi} = f(\xi, \kappa(\xi)), & \xi \in \mathcal{C}, \\ \xi^+ \in g(\xi), & \xi \in \mathcal{D}. \end{cases}$$

$(\mathcal{H})$ satisfies the HBC

HBC guarantee robustness of GAS

Theorem: Robustness to perturbations, measurement noise, and switch uncertainties

There exists a non-trivial perturbation function $\xi \mapsto \mu(\xi)$ such that $\mathcal{A}$ is GAS for the perturbed system $(\mathcal{H}_\mu)$

$$(\mathcal{H}_\mu) \begin{cases} \dot{\xi} = f(\xi, \kappa(\xi)), & \xi \in \mathcal{C}, \\ \xi^+ \in g(\xi), & \xi \in \mathcal{D}. \end{cases}$$

Robustness of GAS is guaranteed by hybrid well-posedness

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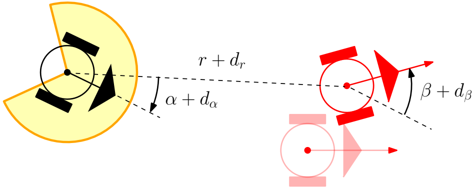
- **Measurement errors in the feedback**



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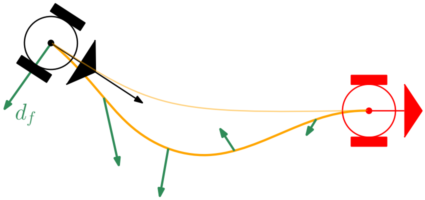
- **Arbitrary external perturbations**



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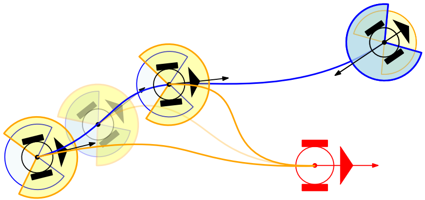
- Measurement errors in the feedback
- Arbitrary external perturbations
- **Uncertainties affecting the switch time**

$$\mathcal{C}_\mu = \{\xi : |\alpha| \le \tfrac{\pi}{2} + \delta + \mu(\xi)\},$$
$$\mathcal{D}_\mu = \{\xi : |\alpha| \le \tfrac{\pi}{2} + \delta - \mu(\xi)\}.$$



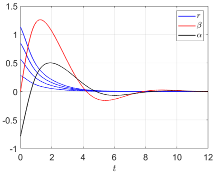
Stability properties in the Cartesian coordinates

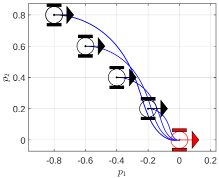
AS in the polar coordinates $z = (r, \beta, \alpha)$ $\Rightarrow$? AS in the Cartesian coordinates $x = (p_1, p_2, \phi)$
The Cartesian state x can be seen as an output of the system

$$\star \begin{cases} \dot{z} &= f(z, \kappa(z)) \\ x &= \gamma(z) \end{cases}$$

- $|x|^2 = p_1^2 + p_2^2 + \phi^2 \overset{x=\gamma(z)}{=} r^2 + \beta^2 \leq |z|^2$
  $\Rightarrow |z(t)| \to 0$ implies $|x(t)| \to 0 \Rightarrow$ **attractivity**

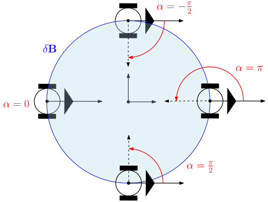
- $|x(0)| \leq \delta_x$ does **NOT** imply $|z(0)| \leq \delta_z$!
  $\Rightarrow$ large $|z(0)|$ can lead to $\beta(t) = -\phi(t)$ overshoot!





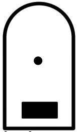
- The coordinate transformation $x = \gamma(z)$ is not a **homeomorphism**!

$$(r, \beta, \alpha) = \gamma^{-1}(\delta\mathbb{B}) = [0, \delta] \times [0, \delta] \times (-\pi, \pi]$$



Stabilization in polar coordinates guarantees convergence in Cartesian coordinates

- Introduction and background
  - Setting & Hybrid Systems & Lyapunov Stability
- Target stabilization
  - Unmanned Ground Vehicles (UGV)
- Recent Extensions
  - Unmanned Surface Vehicles (USV)



Ballaben, R., Astolfi, A., Braun, P., & Zaccarian, L. (2025). *Towards global stabilization of a hovercraft model using hybrid systems and discontinuous feedback laws* [CDC]. Ballaben, R., Astolfi, A., Braun, P., & Zaccarian, L. (2026a). *Hybrid orchestration of onboard cameras for global stabilization of a hovercraft* [Conditionally accepted: IFAC Nonlinear Analysis: Hybrid Systems].